EMQ Model with Maintenance Actions and Warranty Policy for Imperfect Production System

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Abstract: This paper studies the optimal production run length for the economic manufacturing quantity model with maintenance, scrap and rework actions under the imperfect production system where the process state is classified as either in-control or out-of-control. Suppose that a production process starts to produce in an in-control state, however, after a period of time, the process may evolve from this in-control state to an out-of-control state in which more nonconforming items are produced than in the case of an in-control state. Thus we are taking restoration cost, disposal cost and reworked cost for nonconforming items into account in this paper. We show that there exists a unique optimal production run length such that the expected total cost is minimized. In addition, bounds for the optimal production run length are provided to develop the solution procedure. Finally, a numerical example is given to illustrate the results.

Keywords: Inventory, Imperfect Production System Rework, Warranty

I. Introduction

In the classical economic manufacturing quantity (EMQ) models, the production facility is assumed to be failure free and all the items produced are good quality items. However, in real production, the product quality is not always perfect and usually depends on the state of the production process. Recently, significant research has been undertaken to extend the basic EMQ model by relaxing various assumptions so that the model conforms more closely to real-world situations. Considerable attention has been paid to model with an unreliable production facility and to model with production process subject to random deterioration. The EMQ models with a production facility subject to random failure have been studied by Groenevelt et al. [10], Hong et al. [12], Das and Sarkar [5], and Liu and Cao [20] etc. Recently, Chung [4] studied bounds for production lot sizing with machine breakdown. Yeh et al. [30] and Wang and Sheu [29] study the optimal production run length for a deterioration production system in which the products are sold with free minimal repair warranty. Chiu et al. [3] studied the optimal run time problem of economic production quantity (EPQ) model with scrap, reworking of defective items, and stochastic machine breakdowns. Sana [26] investigated an economic production lot size model in an imperfect production system in which the production facility may shift from an in-control state to an out-of-control at any random time. Other related studies include Berg et al. [1], Kelle et al. [15], Jung et al. [14], Chen and Lo [2], Liao et al. [18], and their references.

On the other hand, considerable attention has been paid to EMQ models with deteriorating production process. Deterioration of production is an inherent process in most manufacturing industries because a production system usually deteriorates continuously due to usage or age factors such as corrosion, fatigue and cumulative wear. In the simplest characterization, the process state is classified as either “in-control” or “out-of-control”. Suppose that a production process starts to produce in an in-control state; however, after a period of time, the process may evolve from this “in-control” state to an “out-of-control” state in which more nonconforming items are produced than in the case of an in-control state. Therefore, in much of the inventory analysis literature, the EMQ model has been analyzed under the relaxed assumption that the production process always produces items of acceptable quality. Rosenblatt and Lee [25] initially derive the optimal production run length and study the effect of the system deterioration on the expected cost rate when the maintenance cost is negligible. They assumed that the shift-time from “in-control” to out-of-control” is exponentially distributed and show that the optimal production run length is shorter than that of the classical EMQ model. Porteus [23] has observed similar results. Djamaludin et al. [7] relax the assumption in Porteus [23] that once the production process is in an out-of-control state, there would be a larger fraction of defective items than when the process is in an in-control state. In their model, the product quality is characterized by two failure distributions and a defective item results in more warranty cost. Hariga and Ben-Daya [11] and Kim and Hong [16] studied the RL model by considering the general time required to shift distribution and an optimal production run-length shown to be unique. In order to enhance the reliability of the deteriorating production process, Tseng [28] incorporated a preventive maintenance policy inspection policy into an imperfect EMQ model. Yeh and Chen [31] extended Diamaludin et al. [7] model to determine the optimal lot size and product inspection policy for a deteriorating production system when products are sold with free minimal repair warranty. Recently, Lin and Hou [19] extended Rosenblatt and Lee [25] model to consider the restoration cost for deteriorating production system. Hou [13] considered an EPQ model with imperfect production processes in which the setup cost and process quality are functions of capital expenditure. Chen et al. [2] considered the learning effect of the unit production time on optimal lot size for the imperfect production system with allowable shortage. Yoo et al. [32] proposed a profit-maximizing economic production quantity model that incorporated both imperfect production quality and two-way imperfect inspection. In this paper, the Lin and Hou [19] model is extended under the assumption that an elapsed time until a shift is exponentially distributed and there is a portion to be scrap or reworked for nonconforming items. In addition, we employ a two-state continuous-time Markov chain to describe the deteriorating process of a production system. Thus we are taking restoration cost, disposal cost and reworked cost for nonconforming items into account in this paper. We show that there exists a unique optimal production run length such that the expected total cost is minimized. Furthermore, the bounds for the optimal production run length will be derived and an efficient algorithm to locate the optimal production run length will be developed. Finally, a numerical example is given to illustrate the results and sensitivity analysis is performed to study the effects of changing parameters values on the optimal solution of the system.

II. Mathematical Model

We model the deterioration of the production process by assuming a shift from an "in-control" to an "out-of-control" state. Here, let X denotes the elapsed time of the process in the in-control state and follows an exponential distribution with finite mean $1/\lambda$. We found that early researches like Davis [6] and Epstein [9] have found strong empirical support for such a distribution. As a result, the exponential assumption is often used in the quality control or deteriorating production systems literature (see, e.g. Rosenblatt and Lee [25], Rahim [24], Markis [21], Ouyang and Chang [22], and Lee and Wu [17]). Once the system shifts to the out-of-control state, it stays there until the end of a production run, the system is setup with cost $k > 0$ and is inspected to reveal the state of the system. Inspection and preventive maintenance actions are performed at the end of the production run with a fixed cost v. If the system is an out-of-control state, it can be restored to an in-control state with adjustment or restoration cost $r > 0$ for the next production run. On the other hand, if the process inspected is identified to be in the in-control state, only a preventive maintenance is performed. Therefore, the process is in-control when it starts at the beginning of each production run. Due to variability in the manufacturing process, the quality of items produced may be varied in practice. For simplicity, it is assumed that all the items produced are operational and can be classified as being either conforming or nonconforming depending on whether its performance meets the products' specifications or not. We assumed that an items produced is nonconforming with probability θ_1 (or θ_2) when the production process is an in-control (or out-of-control), where $\theta_1 < \theta_2$.

The nonconforming items will eventually be scrapped and reworked. Among these nonconforming items, a δ portion is considered to be scrap with unit disposal cost of c_s and the other portion can be reworked and repaired with rework cost c_r after the production cycle is over. Suppose that the production rate of the system and the demand rate of the product are p and d, respectively, where $p > d$. Given that the production run length is t, thus, the time duration of a production cycle is $T = pt/d$ and the maximum inventory level is $(p\text{-}d)\,t$. In addition, suppose that the inventory holding cost for carrying an item per unit time is h. Since scrap items are subtracted from inventory, clearly, the expected total inventory holding cost in a production cycle is $\frac{(p-d)t(pt/d)h}{2}$. Thus, the expected inventory holding cost per unit time becomes $(p\text{-}d)\,ht/2$. The restoration cost occurs only when the system is an out-of-control at the end of a production run. Hence, the expected restoration cost per unit time becomes $\frac{rd}{pt}(1-e^{-\lambda t})$. Moreover, to obtain the expected rework cost, we need to compute the expected number of nonconforming items in a production run. Define N as the number of nonconforming items in a production run with length t. Then, we have

$$N = \begin{cases} \theta_1 pt & \text{if } X \geq t \\ \theta_1 pX + \theta_2 p(t-X) & \text{if } X < t \end{cases}$$

Therefore, the expected number of nonconforming items in a production run is

$$\begin{aligned} E(N) &= \int_t^{\infty} \theta_1 pt\lambda e^{-\lambda x}dx \\ &\quad + \int_0^t [\theta_1 px + \theta_2 p(t-x)]\lambda e^{-\lambda x}dx \\ &= \theta_2 pt + p(\theta_1 - \theta_2)\frac{1-e^{-\lambda t}}{\lambda} \end{aligned} \qquad (1)$$

Thus, according to (1), the disposal cost for scrap items and the rework cost for repairable items per unit time is given by

$$\frac{dE(N)[\delta c_s + (1-\delta)c_r]}{pt} = [\delta c_s + (1-\delta)c_r]\left[d\theta_2 + d(\theta_1 - \theta_2)\frac{1-e^{-\lambda t}}{\lambda t}\right] \qquad (2)$$

Let $TC(t)$ be the expected total cost per unit time composed of setup cost, inventory holding cost, restoration cost, disposal cost, and rework cost. Then $TC(t)$ is given as follows.

$$TC(t) = \frac{kd}{pt} + \frac{h(p-d)t}{2} + \frac{rd(1-e^{-\lambda t})}{pt} + [\delta c_s + (1-\delta)c_r]\left[d\theta_2 + d(\theta_1-\theta_2)\frac{1-e^{-\lambda t}}{\lambda t}\right]$$

(3)

So far, we have appropriately established mathematical model that is particularly useful in production-inventory management. Our objective is to find an optimal production run length t^*, which minimizes $TC(t)$ of (3). Furthermore, based on (3), following observations should be mentioned:

(1) When r = 0, $TC(t)$ of (3) reduces to the case that the restoration cost is zero.

(2) When θ_1 = 0, it means that items produced are always conforming when it is in an in-control state.

(3) When $\delta = 0$, it means that the nonconforming items will eventually be reworked. The equation (3) will reduce to the equation (3) in Lin and Hou [19]. Therefore, the proposed model in this paper is an extension of Lin and Hou [19].

Combining the arguments of (1), (2) and (3), the equation (3) will reduce to the equation (1) in Rosenblatt and Lee [25]. Therefore, the proposed model in this paper is also an extension of Rosenblatt and Lee [25].

III. The Optimal Production Run Length

In this section, the unique property of the optimal production run time t^* is obtained and we provide bounds for searching the optimal production run length which minimizes $TC(t)$ of (3). Hence, by differentiating $TC(t)$ with respect t, we have

$$\frac{dTC(t)}{dt} = \frac{1}{t^2}\left[\frac{-kd}{p} + \frac{h(p-d)}{2}t^2 + \beta\left(\lambda t e^{-\lambda t} + e^{-\lambda t} - 1\right)\right]$$

(4)

Where

$$\beta = \left[\frac{rd}{p} + \frac{[\delta c_s + (1-\delta)c_r]d}{\lambda}(\theta_1-\theta_2)\right]$$

(5)

Define $f(t) = t^2\frac{dTC(t)}{dt}$, then

$$f(t) = \frac{-kd}{p} + \frac{h(p-d)}{2}t^2 + \beta\left(\lambda t e^{-\lambda t} + e^{-\lambda t} - 1\right) \quad (6)$$

It is well known that the necessary condition for t^* to be optimal is $f(t^*) = 0$. The following theorem shows that there exists a unique t^* satisfying $f(t^*) = 0$.

Proposition 1. The optimal production run length t^* exists and is unique.

Let $t_1 = \sqrt{\frac{2kd}{ph(p-d)}}$ and $t_2 = \sqrt{\frac{2kd}{p[h(p-d)-\beta\lambda^2]}}$. Then we have the following results.

Proposition 2.

(a) *If* $\beta \le 0$ *then* $0 < t^* \le t_1$.

(b) *If* $\beta > 0$ *then* $t_1 < t^* < t_2$.

Although the optimal run length t^* cannot be expressed in a closed form, it can be obtained through the use of numerical methods. Proposition 1 and Proposition 2 reveal that the bisection method based on the Intermediate Value Theorem is appropriately used to find t^* (see, e.g. Thomas and Finney [27]. Using the bounds, the search procedure for t^* is summarized as follows.

Step 1. Let $\varepsilon > 0$ and compute β, t_1 and t_2.

Step 2. If β > 0, set $t_L = t_1$, $t_U = t_2$. Otherwise, set $t_L = 0$, $t_U = t_1$.

Step 3. Let $t_{opt} = (t_L + t_U)/2$.

Step 4. If $|f(t)| < \varepsilon$, go to step 6. Otherwise, go to step 5.

Step 5. If $|f(t)| > 0$, set $t_U = t_{opt}$.

If $|f(t)| < 0$, set $t_L = t_{opt}$. Then, go to step 3.

Step 6. $t^* = t_{opt}$, compute $TC(t^*)$, and exit the optimal value.

IV. A numerical Example

To illustrate the above solution procedure, let us consider a deteriorating production system with the following data: d = 1000, p = 1500, k = 200, r = 200, c_r = 25, c_s =15, λ = 0.2, h = 2, $\delta = 0.15$ θ_1 = 0.10, θ_2 = 0.75. In addition, we assume that the shift distribution is an exponential distribution with parameter λ = 0.2. Using the above parameters, it can be verified that β is negative. Applying the proposed algorithm, we find t^* = 0.2699 and $TC(t^*)$ = \$3409.876.

V. Conclusions

In this paper, a Markovian EMQ model discussed by Rosenblatt and Lee [25] and Lin and Hou [19] is extended to consider the restoration cost, disposal cost and reworked cost for deteriorating production system with scrap, reworking of the nonconforming items. We show that the optimal production run length t^* is unique and bounded in a finite interval. In addition, an efficient algorithm procedure is provided for deriving t^* easily. Furthermore, the effects of the model parameters on the optimal production run length and on the optimal expected total cost are investigated through a numerical example. The possible extensions of the proposed problem in this paper may be made by considering for products sold with warranty and a shift distribution is Weibull to being considered in the future.

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